

# L: Linguistic Labyrinth

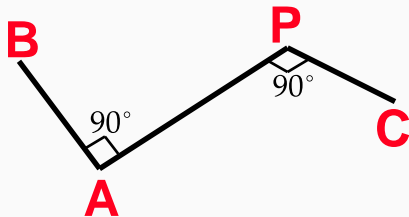
Problem author: Jeroen Op de Beek



**Problem:** Count number of quadruples  $BAPC$  such that  $\angle BAP = 90^\circ$  and  $\angle APC = 90^\circ$ .

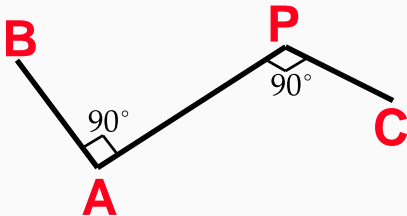


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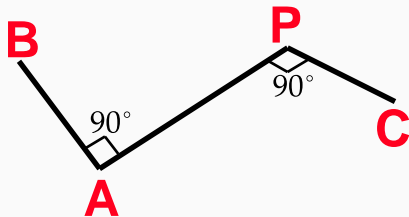
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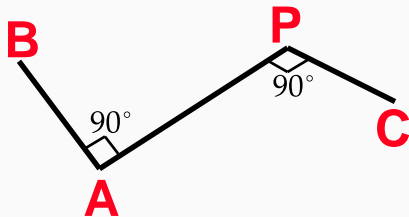


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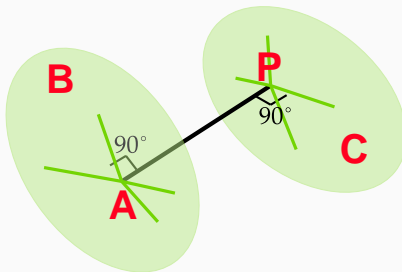
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**Better solution:** Fix  $A$  and  $P$ . Now the choice of  $B$  and  $C$  are independent.

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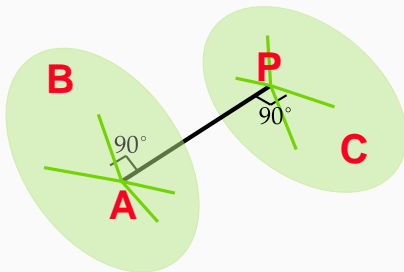
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**Better solution:** Loop over all  $AP$  pairs, count the number of possible  $B$ 's and  $C$ 's, and multiply these counts.

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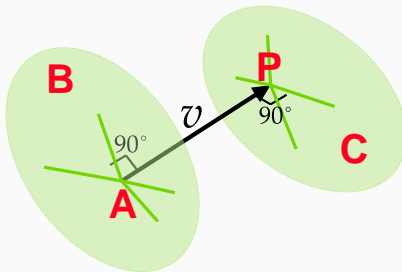


**Better solution:** Loop over all  $AP$  pairs, count the number of possible  $B$ 's and  $C$ 's, and multiply these counts.

**Running time:** Counting  $B$ 's and  $C$ 's takes  $\mathcal{O}(n^3)$  per  $AP$  pair, so the runtime is  $\mathcal{O}(n^9)$  in total, still too slow.

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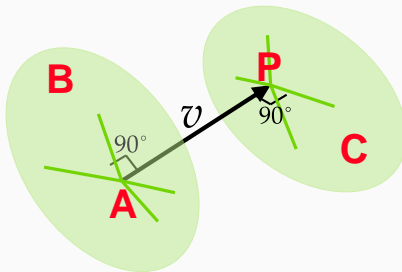


**Best solution:** Let  $v$  be the vector  $P - A$ . Then a point  $B$  is good if and only if  $v \cdot B = v \cdot A$ , and likewise  $C$  is good if and only if  $v \cdot C = v \cdot P$ .



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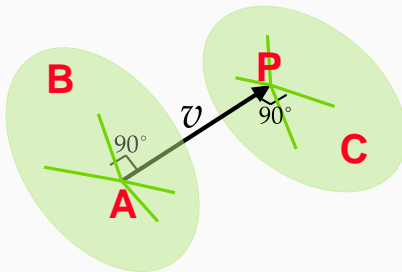


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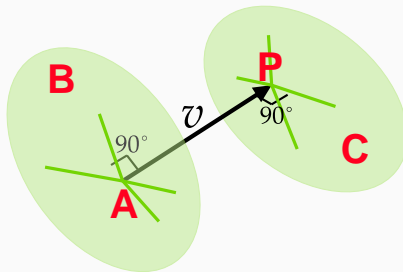
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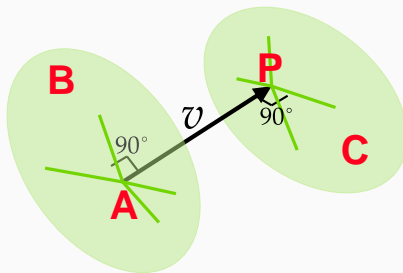
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Statistics: 32 submissions, 0 accepted, 21 unknown